EFFICIENT FULL–WAVE 3D AND 2D WAVEGUIDE EIGENVALUE ANALYSIS BY USING THE DIRECT FD–TD WAVE EQUATION FORMULATION

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ABSTRACT

A modified finite–difference time–domain (FD–TD) formulation based on the direct discretization of the vector wave–equation is applied for the efficient analysis of hybrid–mode waveguiding structures. For both three– (3D) and two–dimensional (2D) waveguide eigenvalue problems, the FD–TD wave equation formulation requires advantageously only one grid, and merely the solution of three coupled equations are necessary. Numerical examples for the resonance frequencies for an inhomogeneously filled waveguide resonator, as well as for the fundamental– and higher–order–mode propagation factors for insulated image guides, shielded coupled dielectric guides, and lateral open dielectric ridge guides demonstrate the efficiency of the method. The theory is verified by comparison with results obtained by other methods.

INTRODUCTION

The finite–difference time–domain (FD–TD) method [1] is well established meanwhile as a versatile numerical tool for solving eigenvalue and scattering problems of a great variety of waveguiding structures [2] – [9]. One of the most attractive features of the method is its flexible applicability for structures with complicated circuit contours. On the the other side, however, a well–known draw–back of the method in its standard formulation is the relatively large amount of memory space and cpu time required, in particular for the full–wave analysis of hybrid mode waveguiding problems in inhomogeneous waveguiding structures. Several advances to reduce the mesh size for special waveguide problems have been reported in the past, therefore, ranging from the application of a two–dimensional FD–TD method for planar circuits [6] to a complex formulation for two–dimensional analysis problems [7], [8].

In this paper, we utilize a different approach, the direct FD–TD wave equation discretization, which has in particular the advantage to be applicable for both two– (2D) and three–dimensional (3D) waveguide eigenvalue problems. This formulation requires only one grid (instead of two displaced grids in the commonly used curl equation approach), and merely the solution of three (instead of six) coupled equations are necessary. Moreover, an actual 2D grid for 2D problems is obtained, i.e. the grid size for these problems is zero in z–direction as long as the waveguide is homogeneous in that direction. A noticeable reduction in cpu time is achieved for both 3D and 2D waveguide problems.

The efficiency of the method is demonstrated at typical examples. Numerical results are presented for the resonance frequencies for an inhomogeneously filled waveguide resonator, as well as for the fundamental– and higher–order–mode propagation factors for insulated image guides, shielded coupled dielectric guides, and lateral dielectric ridge guides (Fig. 1). The theory is verified by comparison with results obtained by other methods.

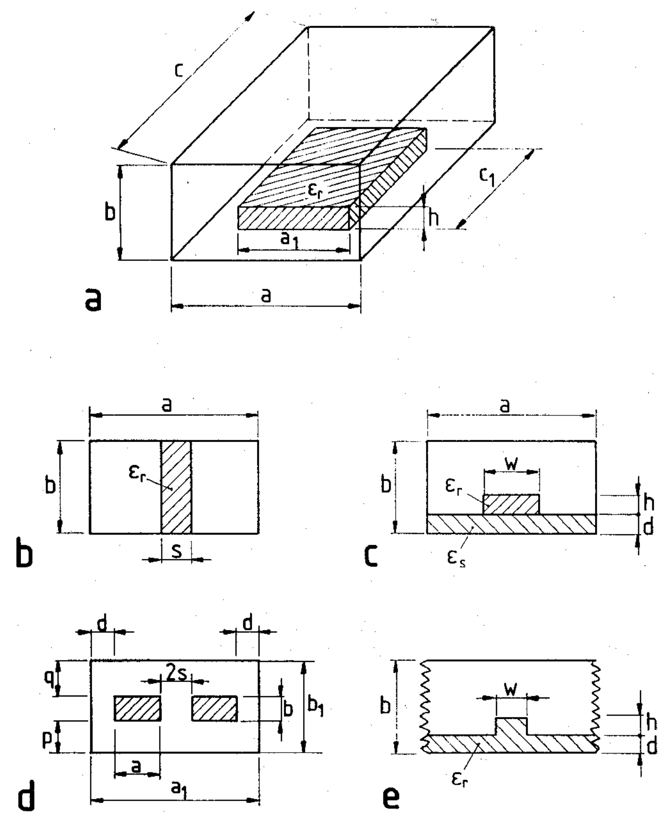

Fig. 1:
The class of 3D or 2D shielded or partially open millimeter–wave and optical waveguide structures investigated with the FD–TD wave equation method.

THEORY

The FD–TD method is usually formulated by discretizing Maxwell's curl equations over a finite volume and approximating the derivatives with centered difference approximations [1] – [8]. This leads to the three–dimensional Yee's mesh [1] in various modifications

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[2] – [5]. In this paper, we discretize directly the vector wave equation for inhomogeneous media

$$\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) , \qquad (1)$$

which becomes in cartesian coordinates (e.g. for E_x)

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\epsilon\ \mu} [\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{\partial}{\partial x} (\frac{\partial E_x}{\partial y} + \frac{\partial E_z}{\partial z})] . \qquad (2)$$

A. Three–dimensional FD–TD wave equation formulation

Following Yee's notation, and assuming for simplicity a uniform discretization of equation (2), a reduced set of FD–TD equations for the electrical field is obtained where only the three electrical field components E_x, E_y, E_z are coupled. We obtain e.g. for E_x (cf. Fig. 2a)

$$\begin{aligned} E_x^{n+1}(i,j,k) = {} & 2E_x^n(i,j,k) - E_x^{n-1}(i,j,k) + \frac{s^2}{\epsilon_{rx}\mu_{rx}} [E_x^n(i,j+1,k) - \\ & 2E_x^n(i,j,k) + E_x^n(i,j-1,k) + E_x^n(i,j,k+1) - 2E_x^n(i,j,k) + \\ & E_x^n(i,j,k-1) - \tfrac{1}{4} [E_y^n(i+1,j+1,k) - E_y^n(i+1,j-1,k) - \\ & E_y^n(i-1,j+1,k) + E_y^n(i-1,j-1,k) + E_z^n(i+1,j,k+1) - \\ & E_z^n(i+1,j,k-1) - E_z^n(i-1,j,k+1) + E_z^n(i-1,j,k-1)]], \end{aligned} \qquad (3)$$

where the stability factor is $s = c\ \Delta t/\Delta l$; c is the velocity of light, and ϵ_{rx}, μ_{rx} are the diagonal elements of the relative permittivity, or permeability tensor, respectively. The condition for stability in free space is $s \leq 1/\sqrt{3}$ [2]. The remaining finite difference equations related to the other two electric field equations can be similarly calculated. The magnetic field, if required, may easily be computed either in the time domain, or, after solving for the electric field in the frequency domain, directly by using Maxwell's second equation.

B. Two–dimensional FD–TD wave equation formulation

For in z–direction homogeneous structures, like in [7], [8], a complex notation is utilized, but in contrast to [7], [8], the phase factors β are directly introduced, rather than using a uniform mesh extension of Δl in z–direction. This yields

$$\vec{E}(z) = \vec{E}(0) e^{-j\beta z};\ \frac{\partial\ \vec{E}^n}{\partial z} = -j\beta\ \vec{E}^n(z);\ \frac{\partial^2 \vec{E}^n}{\partial z^2} = -\beta^2 \vec{E}^n(z). \qquad (4)$$

The two–dimensional formulation for e.g. E_x is then given by (cf. Fig. 2b)

$$\begin{aligned} E_x^{n+1}(i,j) = {} & 2E_x^n(i,j) - E_x^{n-1}(i,j) + \frac{s^2}{\epsilon_{rx}\mu_{rx}} [E_x^n(i,j+1) - \\ & 2E_x^n(i,j) + E_x^n(i,j-1) - \beta^2 \Delta l^2 E_x^n(i,j) - \tfrac{1}{4} [E_y^n(i+1,j+1) - \\ & E_y^n(i+1,j-1) - E_y^n(i-1,j+1) + E_y^n(i-1,j-1)] + \\ & \tfrac{1}{2} j\beta\Delta l\ [E_z^n(i+1,j) - E_z^n(i-1,j)]], \end{aligned} \qquad (5)$$

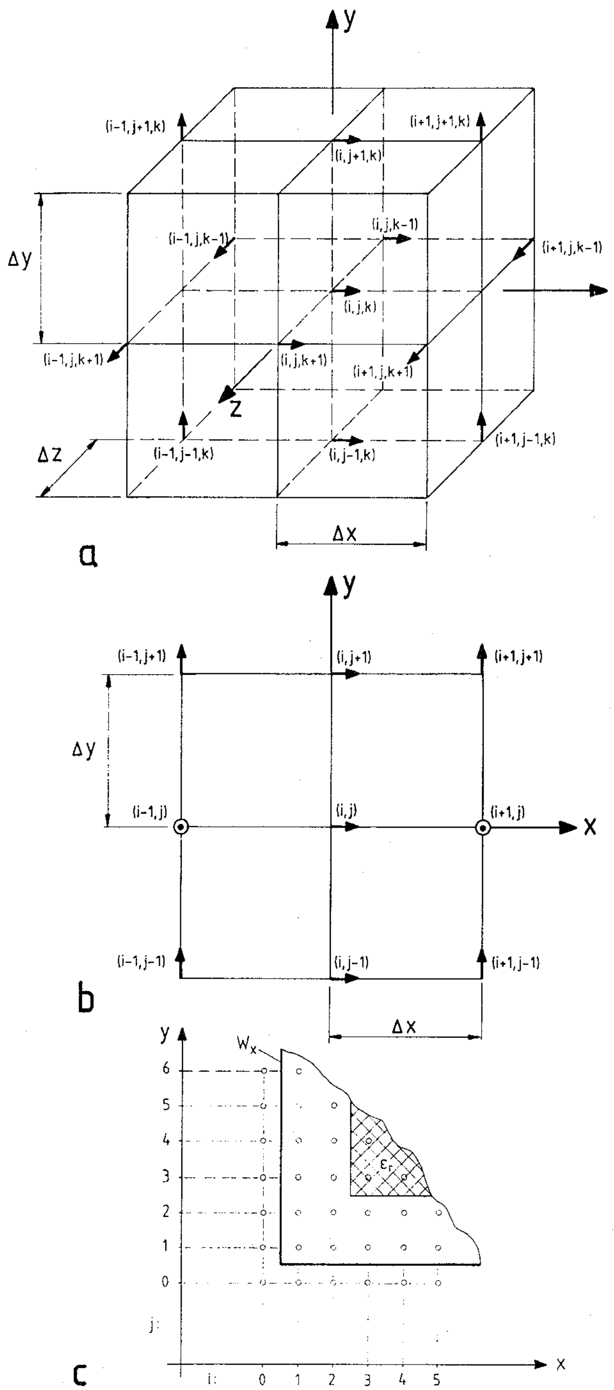

Fig. 2:
Mesh and boundary geometries
a) 3D mesh for the E_x component
b) 2D mesh for the E_x component
c) Electric, magnetic or absorbing boundary location

where $s \leq 1/\sqrt{2}$.

The principal numerical calculation steps for both 3D and 2D problems are similar to those in the conventional FD–TD approach. For 2D problems, however, a propagation factor β has to be selected first. After launching an excitation pulse, waiting until the distribution of the pulse is stable and performing the Fourier transformation, the modal frequencies related to the selected propagation factor are obtained.

C. Absorbing boundary conditions

The electric wall, magnetic wall, and absorbing boundary conditions are assumed to be defined between two mesh nodes (cf. Fig. 2c). The electric and magnetic walls are given in the usual way. Following [9], the FD–TD formulation of the absorbing boundary conditions for the region $x \geq 0$ in terms of the tangential electrical field components in the subregions (0) (outer region), and (1) (mesh region) may be written by

$$E_0^{n+1} = E_1^n + \frac{v_{px}\Delta t - \Delta x}{v_{px}\Delta t + \Delta x}(E_1^{n+1} - E_0^n) , \qquad (6)$$

where v_{px} is the phase velocity in x–direction. The formulation for y is found analogously.

RESULTS

Good agreement between the results of the FD–TD wave equation method for a 3D eigenvalue problem and those calculated by the standard FD–TD and TLM method (own calculations) are demonstrated for the example of a waveguide resonator inhomogeneously filled with dielectric, Fig. 3. For all calculations, the same discretization (10 × 10 × 10) and number of time iterations (N = 2000) is used without any filtering or spectrum estimation techniques. The cpu time for the FD–TD wave equation method is less by about 30% as compared with the standard FD–TD method. A further check of the accuracy was carried out for the simple empty resonator of Fig. 3 by comparison with the results obtained with analytical formulas for the resonant frequency. Although the discretization was relatively coarse ($\Delta l = b/10$), the error was less than one percent.

A comparison with results of the 2D FD–TD method of [8] for the propagation factor of the fundamental mode of the nonradiative guide for different thicknesses of the dielectric slab is shown in Fig. 4. Also very good agreement is stated. The same is true for the shielded insulated image guide, Fig. 5, where comparisons with the 2D FD–TD method of [8] and of the FD–FD method of [10] are presented. The cpu time saving of the FD–TD wave equation method as compared with the already efficient 2D FD–TD method of [8] is about 30%.

As an example where mode–matching results are available, dispersion curves for the even and odd E_{11}^y modes of a shielded coupled dielectric waveguide are presented in Fig. 6. The comparisons with own calculations using the 2D FD–TD method of [8] and by mode–matching (MM) results reported in [11] demonstrate very good agreement. The saving in cpu time is about 31% as compared with [8].

Fig. 7 shows the dispersion curves for different permittivities for the lateral open dielectric ridge guide. The results of the FD–TD method for a relatively low distance of the absorbing boundaries a = 5h (i.e. only 2h distance from the ridge) compare well with those obtained by the FD frequency domain method [10] for the shielded dielectric ridge guide where the lateral shield distance is a = 100h. These curves are considered to verify the absorbing boundary formulations made for the 2D FD–TD wave equation method.

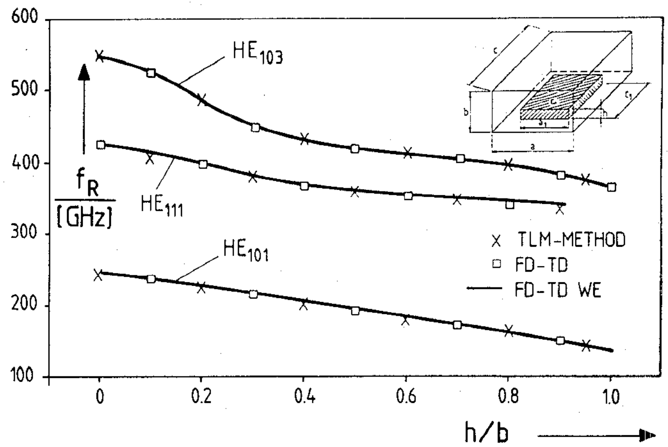

Fig. 3:
Waveguide resonator inhomogeneously filled with dielectric. a_1=c_1=b. Discretization: 10 × 10 × 10. Number of time iterations N_i = 2000. ϵ_r = 4, WR–3 waveguide

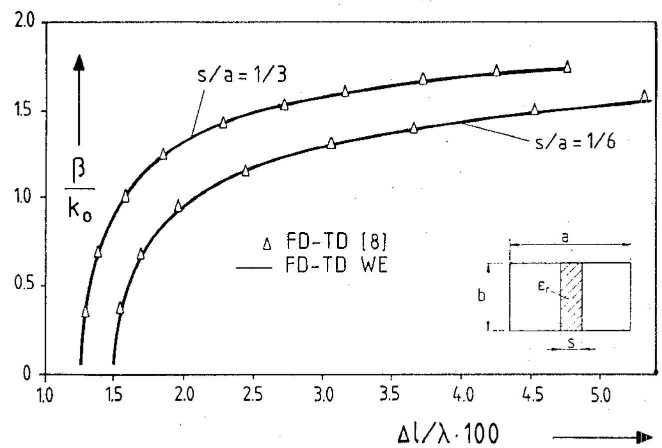

Fig. 4:
Nonradiative image guide. Discretization: 12 × 12, N_i = 2000, ϵ_r = 3.75.

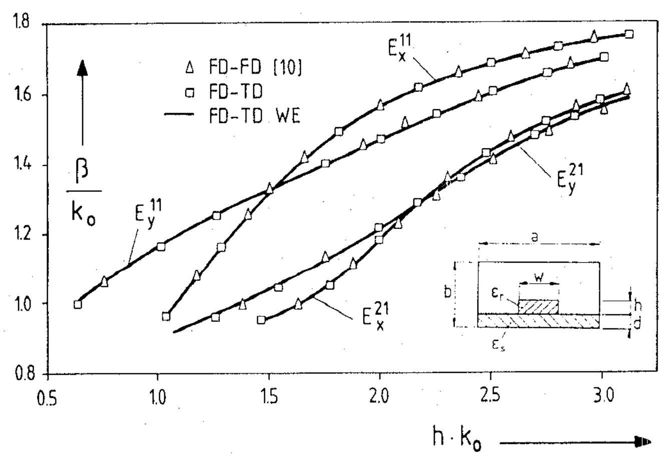

Fig. 5:
Shielded insulated image guide. w/h = 2.25, d/h = 0.5, a/h = 13.5, b/h = 8, ϵ_r = 3.8, ϵ_s = 1.5. Discretization: 54×64. Number of iterations N_i = 1000.

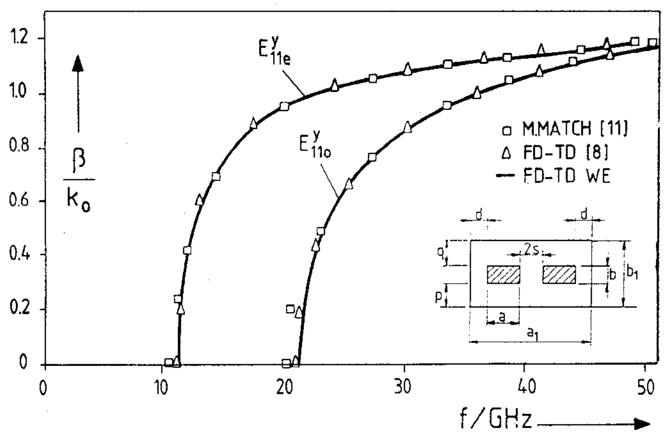

Fig. 6:

Dispersion curves for the even and odd E^y_{11}–modes of a shielded coupled dielectric waveguide. a = 4mm, b = 2mm, s = d = q = p = 1mm. $\epsilon_{r1} = 1.0$, $\epsilon_{r2} = 2.56$ Discretization: 24 × 16. $N_i = 2000$.

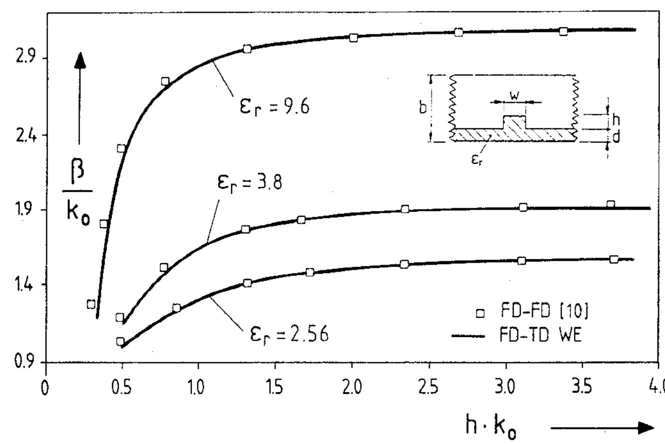

Fig. 7:

Lateral open dielectric ridge guide.w/h = 2, d/h = 1, b/h = 6. Absorbing boundary distance a = 5h. Discretization : 30 × 36. $N_i = 1500$. Comparison with the FD frequency domain method [10] (shielded dielectric ridge guide, lateral shield distance 100h)

CONCLUSION

An efficient full–wave FD–TD formulation based on the direct discretization of the vector wave–equation is proposed for the accurate analysis of 3D and 2D hybrid–mode waveguiding structures. For this formulation, only one grid is required instead of the two displaced grids of the usual approach, and merely the solution of three coupled equations instead of six is necessary. The theory is verified by comparison with results obtained by other methods.

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